Adaptive Control of Nonlinear Systems in the Presence of Actuator Failures

Mahnaz Hashemi

Assistant Professor - Department of Electrical Engineering, Islamic Azad University, Najafabad Branch, Najafabad, Iran
Smart Microgrid Research Center, Najafabad Branch, Islamic Azad University, Najafabad, Iran

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Abstract
This paper presents an adaptive state feedback control scheme for a class of nonlinear systems with unknown parameters, variable control gains and in the presence of unknown time varying actuator failures. The designed controller compensates unknown loss of effectiveness failures as well as unknown time varying stuck failures in actuators. The considered actuator failure can cover most failures that may occur in actuators of the practical systems. The proposed adaptive controller is constructed based on a backstepping design method. Appropriate Lyapunov-Krasovskii functionals are introduced to design new adaptive laws to compensate the unknown actuator failures and unknown parameters. The offered method ensures the asymptotic output tracking and the boundedness of all the closed-loop signals. The proposed design approach is employed for a wing rock control of an aircraft in the presence of time varying actuator failures. The simulation results verify the effectiveness and correctness of the proposed adaptive control method.

Index Terms: Time varying actuator failure, Nonlinear systems, Adaptive control, Backstepping.

کنترل تطبیقی سیستم‌های غیرخطی در حضور خرابی عملگر

مهمان هاشمی

استادیار - دانشکده مهندسی برق، واحد نجف‌آباد، دانشگاه آزاد اسلامی، نجف‌آباد، ایران
مرکز ریسک‌های هوشمند، واحد نجف‌آباد، دانشگاه آزاد اسلامی، نجف‌آباد، ایران
Mahnazhashemi100@gmail.com

خلاصه
در این مقاله، یک کنترل تطبیقی برای کنترل یک کلاس از سیستم‌های غیرخطی در معرض ضرایب‌های نامناسب، بهره کنترلی مناسب و با وجود خرابی عملگر ارائه شده است. کنترل کننده ارائه شده می‌تواند خرابی کاهش کارایی و خرابی قفل شونده در عملکرد یا کاهش عملکرد در نظر گرفته شود قابلیت جبران غلاب خرابی‌های قابل وقوع در سیستم‌های عملی و کاربردی را دارد. کنترل کننده تطبیقی پیشنهادی بر اساس روش کنترلی گام به گام طراحی شده است. در این مقاله، با معرفی توابع لیاپانوف-کراسوکسکی مناسب، قوای تطبیقی جدیدی طراحی شده است که خرابی‌های نامناسب و ضرایب‌های نامناسب را جبران می‌کند. روش کنترلی ارائه شده، تعیین مقداری خروجی و کاربردی تمامی سیستم‌های سیستم حلقه بسته را پس می‌گردد. روش پیشنهادی جهت کنترل بالا هموارا در حضور خرابی‌های منجر به زمان عملگر استفاده شده است. نتایج شبیه‌سازی، کارایی و صحیحی روش کنترلی ارائه شده را نشان می‌دهد.

کلمات کلیدی: خرابی منجر به زمان عملگر، سیستم غیرخطی، کنترل تطبیقی، روش کنترلی گام به گام

Corresponding Author: Mahnaz Hashemi, Assistant Professor - Department of Electrical Engineering, Islamic Azad University, Najafabad Branch, Najafabad, Iran, Mahnazhashemi100@gmail.com
1. Introduction

Actuator failures often cause undesired system behavior and sometimes lead to instability or even catastrophic accidents. The problem of actuator failure compensation is of both practical and theoretical importance, especially for critical systems such as flight control systems. Actuator failure compensation problem is an area of research that has attracted considerable attention in the recent years. So far, varieties of fault compensation methods especially adaptive approaches had been developed [1]-[10]. Adaptive mechanisms show suitable performance in presence of uncertainties in failed actuators. Many valuable researches have been achieved in adaptive actuator failure compensation for linear systems. For example, in [6] direct adaptive state feedback controller scheme was proposed to solve tracking problems for linear systems with unknown system parameters and in the presence of stuck type actuator failures. In [7]-[10], output feedback model reference adaptive controllers were developed for linear systems with unknown parameters in the presence of actuator failures. The considered actuator failure in [7]-[9] were modeled as stuck type and in [10] the actuator failure was modeled to cover both loss of effectiveness and stuck at some unknown fixed values. In [11], the direct adaptive state feedback controller was presented for linear system with actuator failures. The asymptotical stability of all the closed loop signals in [11] was proved despite the presence of loss of effectiveness and stuck type failures in actuators. In [12]-[13], adaptive backstepping method was investigated for nonlinear systems. It was concluded that backstepping’s advantages lies in its flexibility, due to its recursive use of Lyapunov functions and its robustness against unmodeled dynamic of the systems. Some valuable research and practical results have been achieved in actuator failure compensation for nonlinear systems based on the backstepping design method. For example in [14]-[23], adaptive actuator failure compensation schemes were proposed for a class of uncertain nonlinear systems based on the backstepping design method in the form of state feedback [14]-[15], [20]-[23] and output feedback [15]-[19]. The considered actuator failure in [14]-[19] were modeled as stuck at some unknown values. The considered faults in [20]-[23] were modeled to cover both loss of effectiveness and stuck at some unknown fixed values. In [24] adaptive observer was constructed to estimate the fault in a class of nonlinear systems, then a backstepping based active fault tolerant controller was designed for faulty system. In [25], an adaptive fuzzy controller based on the backstepping design method was proposed for a class of nonlinear systems with unknown parameters and actuator failures. In [26], an adaptive fuzzy actuator failure compensator was proposed for a class of uncertain stochastic nonlinear systems in strict feedback form with known control gains. The considered faults in [25]-[26] were modeled to cover both loss of effectiveness and constant stuck failures. The proposed fuzzy adaptive actuator failure compensators in [25] and [26] promised the boundedness of all the signals in the closed loop system; however, the tracking problem was not considered.

In this paper, an adaptive compensator is proposed for a class of nonlinear systems with unknown parameters, unknown control gains and in the presence of actuator failures. The considered actuator failure covers both loss of effectiveness and time varying stuck failures which are uncertain in time, value, and pattern. In other words, during the system operation, it is unknown when, how much and which actuators fail. The proposed adaptive controller in this manuscript is constructed based on the backstepping design method.

The main contributions of this paper, compared with the existing results, are as follows:

1. The control problem is investigated for a class of nonlinear systems with parameter uncertainties and in the presence of unknown actuator failures.
2. The proposed design method does not require a priori knowledge of the bounds of the unknown parameters and actuator failures.
3. The considered time varying actuator failures not only cause the system gain changes but also lead to system uncertainties.
4. The considered unknown time varying actuator failure is more general than the failures considered in the existing results of [14-26].
5. Appropriate Lyapunov-Krasovskii type functionals are introduced to design new adaptive laws with less complexity to compensate the unknown time varying actuator failures as well as uncertainties from unknown parameters.
6. The proposed method ensures the asymptotic output tracking and the boundedness of all the closed loop signals.

The paper is organized as follows. In section 2, the system description is given along with the necessary assumptions. In section 3, the design and analysis of an adaptive actuator failure compensation scheme are explained. In section 4, the actuator failure compensation problem is considered for the F-18 HARV-like wing-rock model to illustrate the effectiveness of the proposed control scheme. Finally, the paper is concluded in section 5.
2. Problem Statement
Consider a class of strict-feedback nonlinear systems in the form:
\[ x_i(t) = x_{i+1}(t) + \theta_i^T F_i(x_i(t)), \quad i = 1, \ldots, n - 1 \]
\[ x_n(t) = \phi_0(x(t)) + \beta^T(x)b(t) + \theta_n^T F_n(x(t)) \]
where \( x_i = [x_1, x_2, \ldots, x_i]^T, \ x = [x_1, x_2, \ldots, x_n]^T, \ u \in \mathbb{R}^m, \ y \in \mathbb{R} \) are the state variables, system input and output, respectively. \( F_i(.) \) and \( \beta(x) = [\beta_1(x(t)), \ldots, \beta_m(x(t))]^T \) are smooth nonlinear function vectors, \( b = \text{diag}(b_1, \ldots, b_m) \) in which \( b_i, i = 1, \ldots, m \) are unknown constant parameters and \( \theta_i, i = 1, \ldots, n \) are unknown constant parameter vectors.

The control objective is to design a state feedback controller for plant (1) in order that all the closed loop signals are bounded and the plant output \( y(t) \) tracks a desired signal \( y_d(t) \) despite the presence of unknown plant parameters, control gains and unknown time varying actuator failures. For this purpose, the following assumptions are considered:

**Assumption 1** The signs of \( b_j, j = 1, 2, \ldots, m \) are known and \( \beta_i(x(t)) \neq 0, j = 1, \ldots, m \).

**Assumption 2** The desired signal \( y_d(t) \) and its first \( n - 1 \) th-order derivatives \( y_d^{(i)}(i = 1, \ldots, n) \) are known, bounded, and piecewise continuous.

The stuck type actuator failures to be considered are modeled as:
\[ u_j(t) = \bar{u}_j(t), \quad t \geq t_j, \quad j = j_1, j_2, \ldots, j_p, \quad 1 \leq p \leq m - 1 \]
\[ \bar{u}_j(t) = \bar{u}_j + \bar{d}_j(t) \]
where \( \bar{u}_j \) is an unknown constant and \( \bar{d}_j(t) \) is given by
\[ \bar{d}_j(t) = \sum_{h=1}^{h} \bar{d}_j(t_h) \]

The failure time instant, \( t_j \), the failure index, \( j \), and the scalar constant, \( \bar{d}_j \), are unknown and the scalar bounded signals \( g_{jl}(t), j = j_1, j_2, \ldots, j_p, l = 1, 2, \ldots, h, h \geq 1 \) are known.

The loss of effectiveness model of the actuator failure to be considered is modeled as
\[ u_j(t) = \rho_j \bar{u}_j(t), \quad \rho_j \in \bar{p}_{j1}, \quad 0 < \rho_j \leq 1, \bar{p}_j \]
where \( \rho_j \) is an unknown constant parameter. For plant (1) with actuator failures (2)-(4), the input vector can be expressed as:
\[ u(t) = \rho v(t) + \delta(\bar{u}(t) - \rho v(t)) \]
where \( v(t) \) is the applied control input that will be designed later. With this description, a general type of actuator failures including loss of effectiveness and time varying stuck failures are considered. Loss of effectiveness can occur due to loss of a part of a control surface, engine malfunction or icing. Variant stuck failures can occur for example due to hydraulic failures that can produce unintended movements in the control surfaces of an aircraft [15]. Table 1 describes different failure situations.

<table>
<thead>
<tr>
<th>Table (1): Failure model</th>
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<tbody>
<tr>
<td>Failure model</td>
</tr>
<tr>
<td>Normal</td>
</tr>
<tr>
<td>stuck</td>
</tr>
<tr>
<td>Loss of effectiveness</td>
</tr>
</tbody>
</table>

For systems in which actuators may fail during the operation of the system, one common way is to use actuator redundancy. In this way, when one actuator fails, some others could compensate for the effect [15].

**Assumption 3:** ([14-26]) In the plant (1) with known plant parameters and failure parameters, if any up to \( m - 1 \) actuators stuck as (2), the others may lose effectiveness as (4), the closed loop systems can still be driven to achieve a desired control objective.

3. Controller Design
In this section, the design procedure of the proposed compensator based on the backstepping method is explained for the system (1). The backstepping design method for system (1) contains \( n \) stages [12]. At each stage, the intermediate control function and updating laws are designed using an appropriate Lyapunov function. To design both the control laws and updating laws, the following state transformation is considered for system (1):
\[ z_1 = x_1 - y_d \]
\[ z_i = x_i - \alpha_{i-1}, \quad i = 2, \ldots, n \]

The transformed system in the new coordination is obtained as:
\[ \dot{z}_1(t) = \dot{z}_2(t) + \alpha_1(t) + \theta_1^T F_1(x_1(t)) - \dot{y}_d(t) \]
\[ \dot{z}_i(t) = \dot{z}_{i+1}(t) + \alpha_i(t) + \theta_i^T F_i(x_{i-1}(t)) - \dot{z}_{i-1}(t) \]
where \( \alpha_i(t) \) is an unknown time-varying function.
\( \dot{z}_n(t) = q_0 \left( x(t) \right) + \beta^T(x)b \delta \check{u}(t) \\
+ \beta^T(x)b \left( 1 - \delta \right) \rho \nu \\
+ \theta_i^T \Gamma_i^{-1} \dot{\theta}_i \)

The detailed design procedure is given as follows.

**Step 1:** In the first step, the \( z_1 \) subsystem is considered and the controller will be designed for this subsystem.

For the \( z_i \) subsystems, the following Lyapunov functions are considered.

\[ V_{z_i} = \frac{1}{2} z_i^T(t) \]  \hspace{1cm} (7)

\[ V_{\theta_i} = \frac{1}{2} \theta_i^T \Gamma_i^{-1} \theta_i \]  \hspace{1cm} (8)

\[ V_i = V_{z_i} + V_{\theta_i} \]  \hspace{1cm} (9)

where \( \dot{\theta}_i = \dot{\theta}_i - \theta_i \) in which \( \dot{\theta}_i \) is the estimate of \( \theta_i \) that will be defined later. Along system (6) and by using (7)-(9), the time derivative of \( V_{z_i} \) becomes

\[ \dot{V}_i = \dot{V}_{z_i} + \dot{V}_{\theta_i} \leq z_i(t) \dot{z}_i(t) + z_i(t) \alpha_i(t) \]

\[ + z_i(t) \theta_i^T \dot{\varphi}_i - z_i(t) \dot{\gamma}_d(t) \]

\[ + \theta_i^T \Gamma_i^{-1} \dot{\theta}_i \]

where

\[ \theta_1 = \theta_i, \varphi_i = F_1(x_i(t)) \]  \hspace{1cm} (11)

Accordingly, the intermediate control input is selected as

\[ \alpha_i(x_i(t)) = -\dot{\theta}_i^T \varphi_i - \frac{1}{2} z_i^2 - \gamma_1 z_i^2 + \dot{\gamma}_d \]  \hspace{1cm} (12)

where \( \gamma_1 \) is a positive constant.

Therefore, the time derivative of \( V_i(t) \) becomes

\[ \dot{V}_i \leq \frac{1}{2} z_i^2 - \gamma_1 z_i^2 - z_i(t) \dot{\theta}_i^T \varphi_i + \theta_i^T \Gamma_i^{-1} \dot{\theta}_i \]  \hspace{1cm} (13)

Accordingly the updating laws for parameters \( \dot{\theta}_i \) is selected as

\[ \dot{\theta}_1 = \Gamma_i z_i \varphi_i \]  \hspace{1cm} (14)

where \( \Gamma_i = \Gamma_i^T > 0 \). By using

\[ -\sigma_1 \dot{\theta}_i^T \dot{\theta}_i \leq \frac{1}{2} \sigma_1 \dot{\theta}_i^T \dot{\theta}_i + \frac{1}{2} \sigma_1 \| \theta_i \|^2 \]

the time derivative of \( V_i(t) \) becomes

\[ \dot{V}_i \leq -\gamma_1 z_i^2 + \frac{1}{2} \dot{\gamma}_d^2 \]  \hspace{1cm} (15)

As can be seen from the above inequality, the time derivative of \( V_i(t) \) is dependent on the boundedness of the \( z_2 \) signal that will be regulated in the following.

**Step 2:** Similar procedures are taken for each step when \( i = 2, \ldots, n - 1 \), as in step 1. The \( z_i \) subsystems for \( i = 2, \ldots, n - 1 \) are considered. The intermediate controllers \( \alpha_{i-1}(t), i = 2, \ldots, n \), are functions of \( \check{z}_{i-1}(t), \theta_{i-1}, \ldots, \theta_{i-1}, \gamma_d, y_d^{(1)} \ldots, y_d^{(n-1)} \), hence the time derivative of \( \alpha_{i-1}(t) \) becomes

\[ \dot{\alpha}_{i-1}(t) = \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} (x_{k+1}(t)) \]

\[ + \frac{\partial \alpha_{i-1}}{\partial \theta_{i-1}} (\check{z}_{i-1}(t)) \]

\[ + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \theta_k} \]

\[ + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(k)}} \]  \hspace{1cm} (16)

The time derivative of \( V_i(t) \) by using (7)-(9) becomes

\[ \dot{V}_i = \dot{V}_{z_i} + \dot{V}_{\theta_i} \leq z_i(t) \dot{z}_i(t) + z_i(t) \alpha_i(t) \]

\[ + z_i(t) \theta_i^T \varphi_i \]

\[ - z_i(t) \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(k)}} + \theta_i^T \Gamma_i^{-1} \dot{\theta}_i \]

where \( \theta_i \) and \( \varphi_i, i = 2, \ldots, n - 1 \), are defined as

\[ \theta_i = \begin{bmatrix} \theta_{i,1}^T, \theta_{i,2}^T, \ldots, \theta_{i,i-1}^T \end{bmatrix} \]  \hspace{1cm} (18)

\[ \varphi_i = \begin{bmatrix} F_{i,1}^T, -\frac{\partial \alpha_{i-1}}{\partial x_{i-1}} F_{i-1,1}^T, \ldots, -\frac{\partial \alpha_{i-1}}{\partial x_1} F_{i-1,1}^T \end{bmatrix} \]  \hspace{1cm} (19)

Therefore, the updating laws and the intermediate controllers are selected as

\[ \dot{\theta}_i = \Gamma_i \dot{z}_i \varphi_i \]  \hspace{1cm} (20)

\[ \alpha_i(t) = -\theta_i^T \varphi_i - z_i - \gamma_i \dot{z}_i \]

\[ + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} x_{k+1}(t) \]

\[ + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \theta_k} \]

\[ + \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial y_d^{(k)}} y_d^{(k)} \]  \hspace{1cm} (21)

where \( \Gamma_i = \Gamma_i^T > 0 \) and \( \gamma_i \) is a positive constant.

Thus, the time derivative of \( V_i(t) \) becomes as follows.

\[ \dot{V}_i \leq -\gamma_i z_i^2 - \frac{1}{2} \ddot{\gamma}_d^2 + \frac{1}{2} \dot{\gamma}_d^2 + \frac{1}{2} \gamma_i \dot{z}_i^2 \]  \hspace{1cm} (22)

It can be seen from the above inequality that the stability of \( z_i \) subsystem in this region is dependent on \( z_{i+1} \), which will be considered in stability analysis of \( z_{i+1} \) subsystem.

**Step 3:** In the final step, the \( z_n \) subsystem is considered. Assuming the knowledge of the uncertainty \( b \) and the actuator failures at time \( t \), the structure of the ideal controller is :
For the time derivative of assumption 3, the above equation always has a Lyapunov function. Consideration assumption 3, the above equation always has a solution. For unknown b, δ and \( \bar{u}(t) \), the adaptive control input designed as

\[
v_j = \frac{k_j(t)v_0}{\beta_j(x)} \quad j = 1, 2, ..., m
\]

where \( v_0 \) is the nominal control to be designed later and \( k_{l,j} \in \mathbb{R} \) is a constant parameter which satisfies

\[
[k_{l,1}, k_{l,2}, ..., k_{l,m}](1 - \delta)p_b[1, ..., 1]^T = 1
\]

with the knowledge of b and actuator failures, \( k_j \) can be achieved from the above equation. Considering assumption 3, the above equation always has a solution.

For unknown \( \theta_0 \) and \( \beta_0 \), the adaptive control input designed as

\[
v_j = \frac{k_j(t)v_0}{\beta_j(x)} \quad j = 1, 2, ..., m
\]

where \( k_j \) is the estimates of \( k \).

For stability analysis, the following Lyapunov functions are considered.

\[
V_u = \sum_{i=1}^{m} (1 - \delta_j) \|b_j\|_{2j} \rho_j \hat{k}_j^2(t)
\]

\[
V_n = V_{\theta_0} + V_{\beta_0} + V_u
\]

where \( \hat{k}_j = k_j - k_j \), in which \( \hat{k}_j \) is the estimates of \( k_j \) and \( V_{\theta_0} \) and \( V_{\beta_0} \) are defined in (7)-(8).

The time derivative of \( V_n(t) \) becomes

\[
\dot{V}_n = V_{\theta_0} + V_{\beta_0} + \dot{V}_u
\]

\[
\leq z_n(t) \phi_0(x) + z_n(t) \sum_{i=1}^{m} (1 - \delta_j) \|b_j\|_{2j} \rho_j \hat{k}_j^2(t)
\]

\[
- \delta_j \rho_j k_j(t)v_0 + z_n(t) \theta_n^T \phi_n
\]

\[
- z_n(t) \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} x_{k+1}(t)
\]

\[
- z_n(t) \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \theta_k} \dot{\theta}_k
\]

\[
- z_n(t) \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial y_{d_{(k-1)}}} y_{d_k}
\]

\[
+ \dot{\theta}_n \Gamma_n^{-1} \tilde{\theta}_n
\]

\[
+ \sum_{i=1}^{m} (1 - \delta_j) \rho_j \|b_j\|_{2j} \hat{k}_j(t) \hat{k}_j
\]

where \( \theta_0 \) and \( \phi_0 \) are defined as

\[
\theta_0 = \left[ \theta_0^T, \theta_{\beta_0}^{(n-1)}T, ..., \theta_{\beta_0}^{(1)}T, \|b_1\|_{2j} \Delta_1 \hat{u}_1, \|b_2\|_{2j} \Delta_2 \hat{u}_2, ..., \|b_m\|_{2j} \Delta_m \hat{u}_m, \|b_1\|_{2j} \Delta_1 \Theta_1, \|b_2\|_{2j} \Delta_2 \Theta_2, ..., \|b_m\|_{2j} \Delta_m \Theta_m \right]T
\]

\[
\phi_0 = \left[ \|b_1\|_{2j} \Delta_1 \Theta_1, ..., \|b_m\|_{2j} \Delta_m \Theta_m \right]T
\]

\[
\beta_2(x) \text{sign}(b_2), ..., \beta_m(x) \text{sign}(b_m), \beta_1(x) \text{sign}(b_1) G_1^T
\]

\[
\Omega_j = [\beta_{\beta_0}^{(n-1)}, \beta_{\beta_0}^{(n-2)}, ..., \beta_{\beta_0}^{(1)}, \beta_{\beta_0}^{(0)}]T
\]

\[
\Omega_{\beta_0} = [\beta_{\beta_0}^{(n-1)}, \beta_{\beta_0}^{(n-2)}, ..., \beta_{\beta_0}^{(1)}, \beta_{\beta_0}^{(0)}]T
\]

The result shows that \( V_n(t) \) is bounded.

Proof: The following Lyapunov function is considered:

\[
V(t) = \sum_{i=1}^{n} V_i
\]

where \( V_i(t) \) for \( i = 1, ..., n \), is defined in (9) and (27). Therefore the time derivative of \( V(t) \) becomes

\[
\dot{V}(t) \leq -\sum_{i=1}^{n} \gamma_i z_i^2
\]

Therefore all the closed loop signals are bounded. It can be seen that \( z_i \in L^2, i = 1, ..., n \) and by considering (6), \( z_i \in L^\infty \) because all the closed loop signals and the derivatives of the desired signal are bounded. Thus \( \lim_{t \to \infty} z_i = 0, i = 1, ..., n \), which implies that \( \lim_{t \to \infty} (y - y_d) = 0 \).

4. Simulation results

In this section, the obtained results are simulated to verify the effectiveness of the proposed method. For this purpose, the actuator failure compensation problem is considered for the F-18 HARV-like wing-
rock model [15, Section 10.1.3]. The aircraft wing model is described as:

\[ x_1(t) = x_2(t) \]
\[ x_2(t) = x_3(t) + \theta_{f_2}^T F_2(t) \]
\[ x_3(t) = \frac{1}{\tau} b^T u(t) - \frac{1}{\tau} x_3(t) \]
\[ y(t) = x_1(t) \]

where the states \( x_1, x_2 \) and \( x_3 \) represent the roll angle, roll rate and aileron deflection angle respectively, \( u(t) \in \mathbb{R}^2 \) is the control input and \( \tau \in \mathbb{R} \) is the aileron time constant which is unknown, \( b \in \mathbb{R}^2 \) and \( \theta_{f_2} \in \mathbb{R}^5 \) are unknown constant vectors and \( F_2(t) = [1, x_1, x_2, x_1 x_2, x_2 x_2]^T \).

The control objective is to track the desired signal \( y_d(t) = 0 \).

For simulation purpose \( \tau = \frac{1}{15}, b = [0.5, 0.2]^T \), and \( \theta_{f_2} = [0, -2.667, 0.86485, -2.9225, 0]^T \).

This simulation example is considered for two actuator failure models in the form of two scenarios.

**Scenario 1** - The failure model in this scenario is considered as

\[ u_1(t) = \begin{cases} (v_1(t), & t < 20 \\ -8, & t \geq 20 \end{cases} \]
\[ u_2(t) = \begin{cases} (v_2(t), & t < 30 \\ 0.5v_2(t), & t \geq 30 \end{cases} \]

The following design parameters are adopted in the simulation:

\[ [x_3(0), x_2(0)]^T = [0.1, -0.1, 0.1]^T, v_1 = v_2 = v_3 = 10, \Gamma_1 = I, \Gamma_3 = 0.11, \theta_{f_2}(0) = [0.5, 0.5, 0.5, 0.5, 0.5, 0.5, 0.5]^T, \theta_{f_3}(0) = 0.5, \theta_3(0) = 0.5, L_1 = 0.5, L_2 = 1 \]

The simulation results are shown in Figs. 1-3. In all of the figures, ‘-’ denotes the time occurrence of the actuator failures.

**Scenario 2** - The considered failure model in this scenario is considered as:

\[ u_1(t) = \begin{cases} (v_1(t), & t < 30 \\ 2 + \sin(t), & t \geq 30 \end{cases} \]
\[ u_2(t) = \begin{cases} (v_2(t), & t < 10 \\ 0.9v_2(t), & t \geq 10 \end{cases} \]

The simulation results are shown in Figs. 4-6.

It can be seen from Figs. 1 and 4 that the asymptotic output tracking is ensured even though there are actuator failures during an operation whose failure time instants, values and patterns are unknown to the adaptive failure compensation controller. Figs. 2 and 5 represent the boundedness of the control inputs and Figs. 3 and 6 show the boundedness of the estimates of the parameters in the control loop system.

As can be seen from Fig. 2, the first input stuck at \( t = 20 \) at constant value and the second input lost 50% of its effectiveness at \( t = 30 \).

For the second scenario, as can be seen from Fig. 5, the first input stuck at \( t = 30 \) at time varying value and the second input lost 10% of its effectiveness at \( t = 10 \).

However, in both scenarios, all the states are asymptotically converged to the origin and all the closed loop signals remain bounded. It can be seen from the results, that the proposed adaptive actuator failure compensator is feasible and effective for the unknown constant and time varying actuator failures of the nonlinear system (1). The above simulation results demonstrate the merits of the proposed design method.

5. Conclusion

In this paper, an adaptive actuator failure compensation scheme is proposed for a class of nonlinear systems with unknown parameters, variable control gains and unknown actuator failures. The considered actuator failure covers both loss of effectiveness and time varying stuck failures which are uncertain in time, value, and pattern. Appropriate Lyapunov-Krasovskii type functionals are introduced to design new adaptive laws to compensate the unknown actuator failures and unknown parameters. The proposed systematic backstepping design method can guarantee global
boundedness of all the closed loop signals in addition to the asymptotic convergence of the system output to the desired signal. Simulation results have been conducted to verify the effectiveness of the proposed method.

References


